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Observables, states and measurements in quantum physics

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Abstract The change in the meaning of observables and states in quantum physics as compared with classical physics is discussed and shown to account for the novel formal properties of quantum observables and states. The special character of observation interactions in quantum physics, in particular, the discontinuous reductions of state vectors during measurements, is shown to be readily understood if the novel meaning of quantum states is taken into account.

Résumé Cet article discute le changement qui intervient dans la signification des observables et des états quand on passe de la physique classique à la physique quantique, et montre que ce changement rend compte des propriétés formelles spécifiques des états et des observables quantiques. On montre, en particulier, que le caractère spécial des interactions liées aux observations en physique quantique, et de la réduction discontinue des vecteurs d'état pendant les mesures, se comprend aisément si la signification nouvelle des états quantiques est prise en compte.

1. Introduction

Even today, more than 50 years after nonrelativistic quantum theory was essentially completed, it is difficult to find an explicit account of the physical meaning of quantum observables, states and measurements in the textbooks. Typically, after discussion of a characteristic experiment, such as the two-slit electron diffraction experiment, the text proceeds directly to a more formal level. Thus, the student is introduced to the novel formal properties of quantum observables, states and measurements whilst retaining essentially classical concepts of these terms. Inevitably, the behaviour of microphysical objects seems 'strange' or 'bizarre'.

The novel formal properties of quantum observables, states and measurements result from the novel meaning of these terms in quantum physics. If these novel concepts are explicitly introduced at an early stage much of the apparent strangeness of 'the strange world of the quantum' can be dispelled. The present essay is an attempt to give an introductory account of these concepts such that their novel formal properties appear as obvious and logical, and not strange. If successful, these explanations should enable consistent and intuitively acceptable answers to be given to the 'puzzling' questions which frequently arise in these areas, for example, 'Where is a microphysical object when it is not observed?', 'Does the superposition principle imply that microphysical objects can be in many different places (or have many different energies, or momenta, etc.) at the same time?', 'Why do reductions of state vectors only occur during measurements?'.

2. Observables

The term, 'observables', refers specifically to the dynamical variables of a system, such as position, momentum, energy and angular momentum. Those quantities, such as mass, charge and spin, which (in nonrelativistic physics at least) remain constant for a given system are not observables in this technical sense of the term.

In classical physics, observables were conceived as intrinsic properties of the observed object, i.e. properties it possessed in itself, independently of the process of observation. This was an idealisation of the actual situation. In fact, all physical observations involved an interaction with the object observed. However, classical theory allowed in principle that all physical actions could be made indefinitely small. Hence, in an observing process, the interaction of observing apparatus and observed object could in principle be made negligibly small in comparison with the quantity being measured. If this were impractical in any particular case, further

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observations of unlimited precision could be made and the results used, in conjunction with the classical laws of motion, to compensate for the effects of interaction in the initial observation.

By contrast, in quantum physics, the interaction between the observed object and the observing apparatus cannot in general be made indefinitely small. There is a finite lower limit to the magnitude of the interaction, determined by Planck's constant, *h*. Furthermore, due to the very small masses of the objects observed (individual molecules, atoms and atomic particles), the interaction will generally be significant in comparison with the quantity being measured. Any additional observations must also involve an irreducible interaction, and hence, cannot be used to give information about the observed object in itself which could be used to compensate for the effects of interaction in the initial observation. Thus, in quantum physics, the interaction of observed object and observing apparatus cannot generally be made negligibly small or compensated for by further observations. As Bohr, in particular, has repeatedly emphasised (Bohr 1963a), in quantum physics the interaction between observed object and observing apparatus forms an inseparable part of what is observed. Consequently, we cannot idealise in quantum physics and regard the results of observation as properties of the observed object alone, independent of the process of observation.

How, then, are we to interpret quantum observables? The observed object and observing apparatus are involved in an irreducible and unanalysable interaction. Since the results of observation derive from this interaction, they must be taken to represent properties of the interaction as a whole, not properties of the observed object alone. Thus, quantum observables are *properties of interactions between the observed object and observing apparatus.* Quantum position, momentum and energy are each properties of a different mode of interaction with observing apparatus. These 'interactional' properties (sometimes also called 'reactive' (Strauss 1972) or 'dispositional' (Popper 1982) properties) have formal properties quite different from classical 'intrinsic' properties.

Classical observables always have a value. This is a consequence of their conception as intrinsic properties of the observed object. It is inconceivable that an object should ever not have a value for any of its intrinsic properties. Intrinsic properties are defining characteristics of an object without which it would not be the kind of object it is. Since all classical observables always have a value, it follows that they all commute, i.e. any selected pair are capable of having a value simultaneously. If quantum observables were also intrinsic properties of the observed object, we would expect them to have the same formal properties as classical observables. Thus, the strangeness felt at the discovery that they do not always have a value and do not all commute is due to failure to recognise their novel conception in quantum physics.

Quantum observables are properties of interactions with observing apparatus. It follows that they can only have a value when observation interactions occur. When no interaction occurs, there cannot be a value for that property of the interaction which is the observable. Thus, quantum observables only have a value at the time of observation. Two quantum observables can only have a value simultaneously if the two corresponding modes of interaction with observing apparatus can occur together. In general, this is not possible in quantum physics, because the modes of interaction corresponding to many observables require mutually exclusive apparatus. For example, as Bohr showed in detail in his famous discussion with Einstein (Bohr 1963b), to measure the position of a microphysical object using a screen with a narrow slit in it requires that the screen be freely movable in relation to the other parts of the apparatus, whereas to use the same apparatus to measure the momentum of the object requires that the screen be rigidly fixed. Thus, position-defining and momentum-defining interactions are mutually exclusive because they require mutually exclusive apparatuses. It follows that the quantum position and momentum observables cannot have a value simultaneously, i.e. they do not commute.

Noncommutation is characteristic of many pairs of quantum observables. This property, and the property of not always having a value, are readily understood if the interactional nature of quantum observables is taken into account.

Although observables are conceived quite differently in classical and quantum physics, the same terms are used to describe them. Thus, the classical terms 'position', 'energy' and 'angular momentum' are used to describe quantum observables. This leads to some confusion. For example, the question of where a microphysical object is between two observations of its position is frequently raised. Because the position observable in quantum physics is called, simply, 'position', and this term in classical physics describes a property possessed by an object at all times, independently of its observation, the expectation that the object must be somewhere between the observations is aroused. It seems baffling to say it is not anywhere, or does not have a position, between the observations.

However, the quantum position observable is not a property of the observed object in itself (e.g. the region of space it occupies at the time), but rather, a property of the interaction of the object with appropriate apparatus, and can only have a value when that interaction occurs. It would, perhaps, be more appropriate to speak of the value of a microphysical object's 'position-defining interaction', or 'momentum-defining interaction', instead of its 'position', or 'momentum'. Then, for example, if we

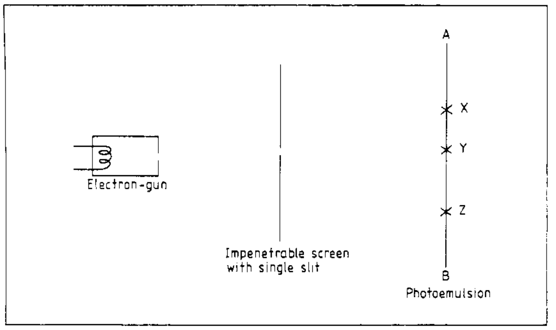

Figure 1 Single slit electron diffraction experiment.

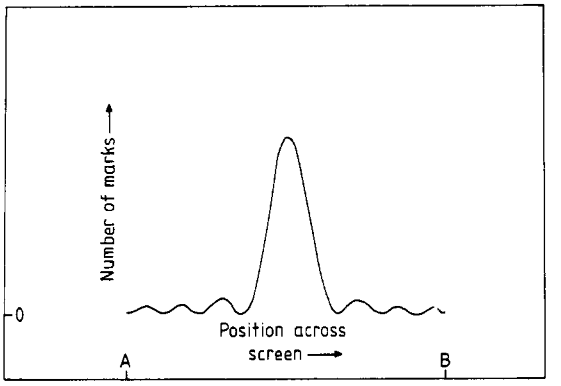

Figure 2 Pattern of results in single slit electron diffraction experiment when a large number of electrons is used.

ask, 'What is the value of a microphysical object's position-defining interaction between position-defining interactions?', the answer, 'In this case it does not have a value', makes sense; where no interaction of the appropriate kind is occurring, the corresponding observable cannot have a value.

3. States

Classical observables were conceived as intrinsic properties of the object observed and a classical state was the totality of the actual values of all classical observables. Thus, in classical physics, the state of an object was the totality of the actual values of its intrinsic properties at a given time.

It is not possible for a quantum state to be the totality of the actual values of all quantum observables. Quantum observables are properties of interactions with observing apparatus which cannot all have a value simultaneously.

The mathematical functions, called 'state vectors', which represent the states of microphysical objects enable us to calculate the probability distributions of results of all possible kinds of observation of the object. This is all we can calculate; the individual results are indeterminate. The probability distribution of results of an observation describes all the possible interactions of the object with the observing apparatus, ascribing a probability to each possibility, i.e. it describes the object's *potentiality* for that particular type of observation interaction. The state vector, from which the probability distributions are calculated, describes the object's potentiality for all possible types of observation interaction. Thus, a quantum state is the totality, not of the actual values, but of the potential values of all quantum observables. The importance of the distinction between actuality and potentiality for the understanding of quantum theory has been particularly emphasised by Fock (1957), Heisenberg (1959), and Bohm (1951). In quantum physics, the state of an object is its potentiality for interacting with observing apparatus to produce particular observational results at a given time. This potentiality has two aspects; a range of possible interactions and a probability associated with each possibility.

Consider the behaviour of microphysical objects, for example, electrons, in the apparatus shown in figure 1. First, consider the behaviour of individual electrons. Each electron is emitted with a definite energy from the electron-gun apparatus. After being diffracted by the narrow slit, it interacts with the photoemulsion, producing a mark at some point in the emulsion between A and B. If the experiment is repeated, with each electron having the same initial energy, they will in general produce marks at different points, such as X, Y, Z, in the emulsion. This result is characteristic of the behaviour of microphysical objects. In general, if the same experiment is repeated, with the same initial conditions, different individual results are recorded. Furthermore, according to quantum theory, this indeterminacy in the behaviour of individual microphysical objects is irreducible; it cannot be explained in terms of our lack of knowledge of finer details of the processes involved. Clearly, the value of the electron's initial energy does not completely determine its interaction with the photoemulsion, but rather, a wide range of possible interactions are left open to it.

If the experiment described above is repeated a large number of times, the marks the electrons produce in the photoemulsion always form a pattern similar to that shown in figure 2. This pattern is completely determined by the initial conditions of the experiment. Thus, whereas the interactions of the individual electrons are indeterminate, the pattern produced by a large number of electrons is completely determined. Clearly, the indeterminacy of the individual interactions is not unrestricted randomness. The pattern in the interactions of a large number of electrons results from a pattern of tendencies in the individual electrons. Though each electron has a wide range of possible interactions open to it, it has a different tendency, or inclination, towards each possibility. These tendencies are described by a weight, assigned to each possibility, corresponding to the probability that the electron will take up that possibility. When a large number of electrons interact, all possibilities are covered, and the frequency with which any particular possibility is taken up indicates the weight of the individual electrons' tendency towards it. The range of possible interactions and the pattern of tendencies together constitute what is here called the potentiality of the electron for interacting with the apparatus. This potentiality is the electron's state at the time of the observation.

The state vector, $\psi(t)$, that represents the state of a microphysical object at time t, is obtained by solving the Schroedinger equation using appropriate initial conditions. For any quantum observable, A, $\psi(t)$ can be expressed (by applying a suitable Fourier transformation) as a linear superposition of states (called 'eigenstates') each of which corresponds to one possible result of measuring observable A at time t. Thus,

$$\psi(t) = c_1\psi_1^A(t) + c_2\psi_2^A(t) + \ldots = \sum_{i=1}^{\infty} c_i\psi_i^A(t). \quad (1)$$

Here, the $\psi_i^A(t)$ are the eigenstates of observable A (assumed for simplicity of presentation to be discrete). Each eigenstate represents one possible state that the observed object may be left in after the measurement of A. The eigenstate $\psi_i^A(t)$ represents the state the object would be left in after the observation interaction leading to the particular observational result, a_i. The coefficients, c_i, are complex numbers from which the probability that any particular observation interaction will occur can be derived. In equation (1), the range of the eigenstates $\psi_i^A(t)$ represents the range of possible interactions between the observed object and the apparatus for measuring observable A. The probability associated with each possibility is given by $|c_i|^2$. Thus, expressed in this way, $\psi(t)$ represents the potentiality of the object for interacting with observing apparatus to produce particular values of observable A.

It is sometimes suggested that superpositions of states, such as that represented by equation (1), imply that, in some mysterious way, microphysical objects can simultaneously possess all the possible values of an observable. Thus, it is sometimes suggested that microphysical objects may be in (or, at least, partly in) more than one place at the same time, or may simultaneously possess many, quite distinct, energies.

Two confusions seem to be involved here. Firstly, quantum observables are not properties of the observed object. They are properties of observation interactions and only have a value when observed. No microphysical object is ever observed to be in two places at the same time or to have two distinct energies at the same time. Secondly, state vectors do not represent the actual values of quantum observables, but rather, their potential values. Thus, superpositions, such as that expressed by equation (1), do not mean that the object simultaneously possesses a multitude of conflicting actual values of a particular observable, but only that a multitude of conflicting potential values of the observable are open to it, any one of which it may take up were it to interact with appropriate apparatus. The state represents the potential interactions of the object, not its actual properties.

When a microphysical object is in an eigenstate of a particular quantum observable, its potentiality for that particular mode of observation interaction consists of just one possibility, which therefore must certainly occur if the observable is measured. Under these circumstances it is often said that the observable has the definite value corresponding to the eigenstate, even though it is not under observation. For example, an electron in a momentum eigenstate is said to have a definite momentum even though it is not being observed. This may appear to contradict the view expressed in section 2 above, according to which quantum observables only have a value at the time of observation. However, it is clear that this a special sense of an observable having a value, which cannot be used generally, for objects whose states are arbitrary superpositions of eigenstates. When an observable is said to have a value in this special sense, what is meant is that, *if* the object were to interact with appropriate apparatus, one particular observational result would, with certainty, be produced. Even when a microphysical object is in an eigenstate of a particular observable, the actual measurement of that observable must still involve an unavoidable and unanalysable interaction with the apparatus. What is special in this case is not that no observation interaction occurs, but rather, that the final observational result can be predicted with certainty. Thus, the special, conditional sense in which an object in an eigenstate of an observable can be said to have a value for that observable, even though it is unobserved, does not contradict the interactional conception of quantum observables. In the fundamental sense explained in section 2, it remains true that the observable only actually has a value when it is observed, i.e. when the expected observation interaction actually occurs.

Mathematically, quantum state vectors have the form of 'wave functions'. However, it is only exceptionally, when individual microphysical objects alone are being considered, that these functions can be interpreted as representing waves in ordinary three-dimensional space. Even in this case the amplitudes of the 'waves' are represented by complex numbers and they behave in ways that would not be expected of classical waves. For example, during a measurement process the quantum 'wave' undergoes an instantaneous change of form unlike anything encountered in classical physics. In general, a system of N microphysical objects will be described by a complex, $(3 \times N)$-dimensional 'wave' which has no simple representation in ordinary space. Thus, state vectors do not represent actual waves in ordinary three-dimensional space, but rather, probability waves, i.e. wave-like potentialities for interacting with our observing instruments. Probability waves evolve in a complex, infinite-dimensional vector space, called Hilbert space. Hilbert space is real. It is not merely an abstract, mathematical construction existing solely in the human mind, but rather, is an aspect of physical reality itself. Classical physics identified reality with actuality. In quantum physics, this concept is extended to include two aspects; actuality and potentiality. Hilbert space is the space of potentialities, i.e. the weighted possibilities alternative to what actually occurs, which encompasses and goes beyond the ordinary three-dimensional space of actualities.

4. Measurements

In classical physics, measurements were conceived as the passive registration of properties already there in the observed object. However, as shown in section 2 above, this idealisation is not possible in quantum physics. Quantum measurements involve unavoidable and irreducible interactions between the observed object and observing apparatus, and quantum observables are properties of these interactions.

In quantum physics, observation interactions are a special class of physical interaction. The axioms of quantum theory distinguish two different kinds of physical interaction. These are *ordinary physical interactions* (described in Axiom 4 of the simplified presentation of the quantum formalism given by Jammer (1974a)) and *observation interactions*, i.e. interactions with observing apparatus leading to particular observational results (described in Axioms 2, 3 and 5 of Jammer's presentation).

During ordinary physical interactions, such as the interaction between an electron and an electric or magnetic field, the states of microphysical objects change continuously and deterministically in accordance with the Schoedinger equation. By contrast, during observation interactions they change discontinuously and indeterministically in accordance with a probabilistic law. If the state of a microphysical object immediately prior to the measurement of an observable, A, is represented as in equation (1) above, then, during the observation interaction, this state changes abruptly from $\psi(t)$ to one of the eigenstates $\psi_i^A(t)$, say $\psi_a^A(t)$. This change is represented formally by the discontinuous transition $\psi(t) \rightarrow \psi_a^A(t)$. Which particular interaction will occur is not predetermined, but the probability that the interaction described by the transition $\psi(t) \rightarrow \psi_a^A(t)$ will occur is given by $|c_a|^2$.

In the special case in which the state of the observed object is initially already an eigenstate of the observable being observed, the result of the observation is predetermined by the initial state of the object, and consequently, could be taken to describe a property of the observed object in itself and not a property of the interaction between the object and the apparatus. However, this interpretation could not be maintained generally for all initial states of the observed object, which are generally superpositions of eigenstates to which no predetermined observational result corresponds. Hence, it is preferable to regard the result of observation even in this special case as describing an interaction of the observed object and the observing apparatus described, in this case, by a null transition, $\psi_a^A(t) \rightarrow \psi_a^A(t)$. The discontinuous changes of state vectors during measurements, generally referred to as the 'reduction' or 'collapse' of state vectors, are instantaneous events. Quantum theory offers no physical explanation, in the sense of a derivation from more general physical laws, of how these events occur; they are simply postulated in the axioms of the theory. In particular, it is not possible to explain these events in terms of the special nature of observing apparatus.

All observing apparatus has certain special physical properties by virtue of which it is capable of amplifying the effects of microphysical observation interactions to yield macroscopic observational results corresponding to a particular quantum observable. These properties vary from one form of apparatus to another but always seem to involve a component which, before the observation interaction, is in a metastable state. Photoemulsions, supersaturated vapours in cloud chambers, superheated liquids in bubble chambers and the charged vacuum chambers of Geiger counters are examples of these metastable components of observing apparatus. The discontinuous state changes that occur during observation interactions trigger irreversible, avalanche processes in this component of the apparatus which amplify the observation interaction to yield a macroscopic result.

However, though such metastable components may be essential to the functioning of observing apparatus, they are not capable of physically explaining the discontinuous state changes that occur during observation interactions. Several physicists have attempted such explanations, trying to deduce the discontinuous changes during measurements from an application of the ordinary law of continuous state change to the special case of apparatus in a metastable state. However, these attempts have not been successful (for a review, see, d'Espagnat (1976a)). Thus, though the presence of observing apparatus is a necessary condition for the occurrence of these events, the discontinuous reductions of state vectors during measurements cannot be physically explained in terms of the special nature of observing apparatus. The reductions of state vectors during quantum measurements are instantaneous, unanalysable events for which no detailed physical explanation can be given.

Some physicists, most notably Wigner (1967a) and Everett III (1973), find it hard to accept the two different kinds of physical interaction postulated by quantum theory. Wigner has described it as a 'strange dualism' (Wigner 1967b). It seems strange for two reasons: Firstly, the discontinuous, indeterministic changes of state that occur during observation interactions are unlike anything that occurs in classical physics, where waves always change their form in a continuous way, and secondly, these discontinuous changes only occur during observation interactions and never during ordinary physical interactions. However, if the novel meaning of quantum observables and states is taken into account, the special character of observation interactions no longer seems so mysterious.

Consider an experiment to observe the position of a microphysical object using a photoemulsion. When we represent the object arriving at the photoemulsion by a wavefront extending over the whole surface of the emulsion, and yet subsequently observe it to produce a mark in only one particular small region, it may seem that the interaction has involved a mysterious collapse, or sudden concentration, of a widely extended wave into a small area. Imagine a plane wavefront approaching a row of swimmers down a swimming bath. The wavefront extends across the full width of the bath, and we would expect each swimmer in the row to experience a slight jolt as the wave reaches the row. It would be very strange if, just as it reached the row of swimmers, the wavefront were to instantaneously collapse and, concentrating all its energy at one point, eject one of the swimmers out of the bath, leaving no disturbance anywhere else. Such a situation may seem analogous to the collapse of a state vector during a measurement. However, this view assumes that the state vector represents an actual wave in three-dimensional space. In fact, as we have seen, the state vector represents the microphysical object's potentiality for interacting with observing apparatus to produce particular observational results; it represents all the possible interactions of the object and their respective probabilities. The wave extending over the surface of the photoemulsion indicates that it is possible for the object to interact anywhere on that surface, and the intensity of the wave at any point gives the probability of the interaction occurring there. When the object interacts at some particular point, it takes up one of these possibilities, and its potentiality changes abruptly. In particular, it will in general no longer be possible for it to interact at all other points of the surface; in taking up one possibility, it excludes the others. Thus, the 'collapse' of a state vector does not represent a mysterious modification of an actual wave, but rather, a logical reduction of the set of potential interactions open to the observed object due to taking up of one of them.

Why do discontinuous reductions of state vectors only occur during observation interactions and not during all physical interactions generally? This question arises because classical physics makes no distinction between observation interactions and ordinary physical interactions. Classical states change in essentially the same way regardless of whether the object is interacting with observing apparatus or with any other physical object. Thus, from a classical point of view, it seems strange that the state of a microphysical object changes in a unique way during observation interactions. However, as shown in section 3 above, the term 'state' does not have the same meaning in quantum physics as in classical physics. The state of a microphysical object represents its potential observation interactions. The discontinuous reduction of this state during an observation represents the actual occurrence of one of these potential interactions. Observation interactions are interactions *with observing apparatus* leading to particular observational results. Clearly, such interactions can only actually occur in the presence of observing apparatus.

When one microphysical object interacts with another, or with an external field of force, though its potentiality for observation interactions (i.e. its state) will generally change as a result of the interaction, it cannot take up any of these potential interactions. It is only observing apparatus that is capable of amplifying the effects of microphysical observation interactions to yield macroscopic observational results, and hence, it is only in interaction with such apparatus that these interactions can occur. Thus, discontinuous reductions of state vectors only occur during interactions with observing apparatus because, given the physical meaning of these reductions, it is only during such interactions that they *can* occur.

There is a particular class of experiments in quantum physics which may appear to invalidate the interactional conception of quantum observables by suggesting that it is possible to measure the value of a quantum observable for a system without in any way interacting with it. These are experiments of the 'Einstein–Podolsky–Rosen (EPR) type', named after the authors of the famous paper which first drew attention to the possibility of this type of experiment (Einstein *et al* 1935). In the typical EPR experiment, two microphysical objects, 1 and 2, interact for a short time and then separate to a large distance so that all interaction between them by known forces effectively ceases. According to quantum theory, at any time, t, after their interaction has ceased the combined system can be represented by a state vector having the general form,

$$\psi^{12}(t) = \sum_{i=1}^{\infty} \psi_i^1(t)\psi_i^2(t). \tag{2}$$

Here, the $\psi_i^1(t)$, describing object 1, may represent eigenstates (again assumed, for simplicity of presentation, to be discrete) of a quantum observable, A. Then, as shown in detail by Einstein, Podolsky and Rosen in their original paper, the $\psi_i^2(t)$, describing object 2, will also represent eigenstates of A.

Suppose that, at some time, t, after the two objects have ceased to interact and are widely separated, object 1 is made to interact with apparatus for measuring A. During the observation interaction, the state of object 1 will undergo a discontinuous reduction to one of the eigenstates of A, say $\psi_N^1(t)$, giving rise to the particular observational result, a_N^1. At the same time, due to the correlation of the states of the two objects implied by equation (2), the state of object 2 will change abruptly to the eigenstate $\psi_N^2(t)$, corresponding to the observational result, a_N^2. Thus, the observation of object 1 in one region of space immediately affects the state of object 2 in a distant region, even though, by hypothesis, there is no interaction between them occurring at the time. A detailed discussion of the immediate, long-range correlations between quantum states implied by EPR type experiments is beyond the scope of the present essay. Thorough discussions of these phenomena and their possible interpretations have been given by d'Espagnat (1976b) and Jammer (1974b).

Do experiments of the EPR type invalidate the interactional conception of quantum observables? As a result of the observation of object 1, object 2 is thrown into an eigenstate of observable A. However, it is only in the special, conditional sense explained in section 3 above that an object in such a state can be said to have the value corresponding to the eigenstate. In the more fundamental sense explained in section 2, even when the object is in an eigenstate of the observable, that observable only actually has a value when an observation interaction occurs. Thus, object 2 only has the definite value a_N^2 for observable A, in the fundamental sense, when it is observed, i.e. when it interacts with appropriate apparatus.

What is novel about EPR type experiments is that they enable us to influence the *state* of a distant object without in any way interacting with it. However, from the point of view of the physical interpretation of quantum observables, they present no essential novelties. It remains true in the fundamental sense that the distant object only actually has a value for any quantum observable when it interacts with observing apparatus. The observable is a property of the observation interaction and can only have a value when an interaction occurs.

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