It is not essential, although certainly convenient, to account classically for the electric fields and the EMF in terms of potentials. An EMF is defined as the work done on a carrier with unit charge on traversing the circuit, in the present case (with a stationary pick-up loop) by the electric forces. If the concept of potential is introduced it is instructive to draw attention to Ehrenfest's theorem (see, e.g. Schiff 1968) according to which a classical force, or corresponding field, is best regarded as an average of the effects of the potential over the wavepacket representing the carrier. From a quantum-mechanical point of view, potential is a more fundamental concept than a force field, as also shown by the Aharonov–Bohm Effect (see, e.g. Chambers 1960).

References

Chambers R G 1960 *Phys. Rev. Lett.* **5** 3–5
Chute F S and Vermeulen F E 1982 *Electron. Wireless World* **88** August 79–80
Gough W and Richards J P G 1986 *Eur. J. Phys.* 7 195–7
Morton N 1980 *Eur. J. Phys.* **1** 138–43
Schiff L I 1968 *Quantum Mechanics* 3rd edn (New York: McGraw-Hill) pp 28–30, 179

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Microphysical objects as 'potentiality waves'

The approach to nonrelativistic quantum mechanics based on the concept of potentiality (Heisenberg 1959, Fock 1957), which I reviewed recently (Villars 1984), suggests the expression 'potentiality wave' as a novel informal term for describing the nature of microphysical objects. This can be used to give simple, intuitive accounts of quantum phenomena without recourse to the 'ambiguities' (see p 154 of Heisenberg (1959)) of the wave–particle duality. As the expression was not introduced explicitly in the previous essay, I would like to offer the following brief explanations.

(i) Quantum mechanics distinguishes two, fundamentally different, kinds of physical interaction: *ordinary physical interactions*, e.g. the interaction between an electron and an electric or magnetic field; and *observation interactions*, i.e. interactions with observing instruments leading to the production of definite observational results, e.g. position measurements.

(ii) A quantum state function describes the *potentiality* of an object for interacting with observing instruments to produce particular observational results, i.e. its potential observation interactions (Villars 1984).

(iii) Quantum state functions are vectors in Hilbert space. Hence, microphysical objects fundamentally exist and evolve in Hilbert space not ordinary three-dimensional space.

(iv) When a microphysical object is unobserved, the state vector describing it evolves in accordance with the time-dependent Schrödinger equation. This has the mathematical form of a wave equation. Hence, the object evolves as a wave in Hilbert space. This is a *wave of potential observation interactions* or, more briefly, a *potentiality wave.*

(v) Potentiality waves in Hilbert space can always be 'projected' into ordinary space as a distribution of potential position-defining interactions (Strauss 1972). As the potentiality wave evolves in Hilbert space, the projected distribution of potential position-defining interactions evolves in ordinary space. Sometimes, the evolving distribution in ordinary space can be represented as an extended wave train. In this case, the potentiality wave gives rise to diffraction and interference effects similar to those produced by classical waves. At other times, the projected distribution has the form of a compact wave packet and the potentiality wave produces phenomena, such as 'particle tracks', which strongly suggest the idea of classical particles. However, in general, the projection of a potentiality wave into ordinary space has no simple interpretation. Thus, the so called 'wave–particle duality' is a relatively superficial characterisation. Both wave-like and particle-like effects are produced by the same underlying entities.

(vi) Potentiality waves differ from *probability waves* in that the latter are usually conceived as abstract, mathematical devices which represent, in a statistical way, the behaviour of *particles.* By contrast, potentiality waves, as their more concrete name suggests, are conceived as physically real waves which exist in their own right, not merely as representations of the behaviour of particles. Microphysical objects are not particles 'guided' in some mysterious way by 'waves of probability', but rather, microphysical objects *are* waves of potential observation interactions.

(vii) The difference between potentiality waves and classical waves is revealed most clearly during quantum measurements. During measurements, potentiality waves change discontinuously in a way that would be difficult to understand if they were waves of actual disturbance in the classical sense. However, if the wave is a potentiality wave, i.e. a wave of potential observation interactions, these discontinuous changes can be readily understood as corresponding to the *actual occurrence* of one of the potential interactions (Villars 1984, 1986).

(viii) Finally, as an illustration of the use of the potentiality wave terminology, consider the well known two-slit electron diffraction experiment. In a typical account, e.g. that given by Feynman (1965), the particle idea is used to explain how individual electrons produce results at discrete places on the detection screen and the complementary wave idea is used to explain the interference pattern produced when many electrons pass through the apparatus. If the potentiality wave terminology is adopted, the experiment can be described, without having to switch between wave and particle pictures, as follows. A potentiality wave, initially representable as a three-dimensional plane wave, enters the apparatus, is diffracted by the double slit and arrives at the detection screen. Here, it actualises one of its potential position-defining interactions, producing a result at some discrete point on the screen. Thus, an interference pattern is produced when many electrons are used because each potentiality wave evolves like an extended wave in ordinary space. On the other hand, individual electrons produce results at discrete points because potentiality waves are waves of *potential effects* at each point on the screen, some one of which is actualised in each case.

References

Feynman R P, Leighton R B and Sands M 1965 *Lectures on Physics* vol. 3 (New York: Addison-Wesley) ch 1.

Fock V A 1957 *Czech. J. Phys.* **7** 643

Heisenberg W 1959 *Physics and Philosophy* (London: Allen and Unwin) pp 42, 53, 139, 156, 159, 160

Strauss M 1972 *Modern Physics and its Philosophy* (Dordrecht: Reidel) p 102

Villars C N 1984 *Eur. J. Phys.* **5** 177

—— 1986 *Phys. Educ.* **21** 232

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On the adjacent coefficients of certain real orthogonal polynomials

The use of special functions in quantum mechanics is well known (Pauling and Wilson 1935). A simple glance at the tables of the familiar real orthogonal polynomials named after Hermite, Legendre and Laguerre (Rainville 1960) shows that the adjacent coefficients in them alternate in sign. If $n \geqslant 0$ is the degree of the polynomial, then the Hermite and Legendre polynomials contain only those and all those powers of x which are congruent to n(mod 2); all the powers of x right from 0 up to n are present in the Laguerre polynomials. The purpose of this Letter is to explain these facts using the properties of their zeros (Rainville 1960, Szegö 1975) and Descartes' rule of signs (Labarre 1961). These results can be easily and immediately generalised.

All the zeros of real orthogonal polynomials are real, distinct and are located in the interior of the interval of orthogonality (Rainville 1960, Szegö 1975). Moreover (Szegö 1975), real orthogonal polynomials $p_n(x)$ associated with an *even* weight function are even or odd polynomials according to whether n is even or odd($n=0, 1, 2, 3, \ldots$), when the interval of orthogonality is *symmetric* with respect to the origin. Szegö (1975) has stated: 'It [$p_n(x)$] *can contain* only those powers of x which are congruent to n(mod 2).' We shall prove that $p_n(x)$ *does contain* all the powers of x which are congruent to n(mod 2).

Now the Hermite polynomials are associated with the even weight function $\exp(-x^2)$ and have the interval of orthogonality $[-\infty, \infty]$. The Legendre polynomials are orthogonal with respect to an even weight function which is simply unity, on the interval $[-1, 1]$ (Rainville 1960). Hence the Hermite and Legendre polynomials of degree n are even (odd) polynomials in x when n is even (odd), $n \geqslant 0$. Let R stand for H or P. Then

$$R_{2n}(x)=\sum_{m=0}^{n} a_{2m}x^{2m} \qquad n\geqslant 1 \qquad (1)$$

since $R_{2n}(x)$ is an even polynomial in x. If $R_{2n}(x)$ has N positive real (>0) zeros, it also has N negative real (<0) zeros, since $R_{2n}(-x)=R_{2n}(x)$. Let $N_z \geqslant 0$ be the number of coincident zeros of $R_{2n}(x)$ occurring at $x=0$. Since $R_{2n}(x)$ is a polynomial in x of degree $2n$ and has $2n$ real zeros (Rainville 1960)

$$N_z=2(n-N) \qquad (2)$$

i.e. N_z is even. As $R_{2n}(x)$ has distinct zeros (Rainville 1960), $N_z \geqslant 2$ is impossible. Hence $N_z=0$ and $N=n$. Thus $R_{2n}(x)$ has n positive zeros. Descartes' rule of signs (Labarre 1961) states that a real polynomial $f_n(x)$, $n \geqslant 1$, cannot have a greater number of positive real zeros than it has variations in sign. Hence there must be at least n variations in sign in the coefficients of $R_{2n}(x)$. Since the maximum number of terms in the right-hand side of equation (1) can be $n+1$ only, the maximum number of variations in sign in the coefficients of $R_{2n}(x)$ can be only n. We can have n variations in sign in the coefficients only when every a_{2m} is non-zero and the signs alternate throughout from a_{2n} to a_0. Hence, in equation (1), none of a_{2m} can be zero and no pair of adjacent coefficients can have the same sign.

In the case of $R_{2p+1}(x)$, an odd polynomial in x, it is clear that $x=0$ is a zero of $R_{2p+1}(x)$, $p \geqslant 0$. By an argument similar to the one used above, one can see that

$$R_{2p+1}(x)=x\sum_{s=0}^{p} a_{2s+1}x^{2s} \qquad p\geqslant 0 \qquad (3)$$

has no missing powers of x and that the coefficients a_{2s+1} alternate in sign throughout.

Our results for the Hermite and Legendre polynomials can be easily and immediately generalised: the adjacent coefficients of real orthogonal polynomials associated with an even weight function alternate in sign if the interval of orthogonality is symmetric with respect to the origin. The polynomial of degree $n(\geqslant 0)$ contains only those